conclusions

- in the SM the two different definitions of the running fermion mass could be introduced: $m(\mu)$ and $m_Y(\mu)$ in contrast to QCD where $m(\mu)$ is the most natural definition
- the relationships between $m_Y(\mu)$ and M have much better convergency properties then those between $m(\mu)$ and M
- in particular, in the case of the bottom quark with the inclusion of EW corrections it is $m_{Y,b}(\mu)$ rather than $m_b(\mu)$ to be used to describe the mass effects
- for the current value of the Higgs mass around $\sim$ 126GeV the EW corrections almost compensate the QCD contribution, reducing the uncertainty in $m_t(M_t) - M_t$ down to the order of 100GeV
- the shift $m_t(M_t) - M_t$ is of phenomenological importance for the combination of different determinations of the top quark mass in ongoing experiments
- it is necessary to determine value of $\overline{\text{MS}}$ mass m_t with the inclusion of NLO electroweak corrections to the total cross-section of the top pair production; this value can then be transformed to the pole mass M_t
- the question of the electroweak vacuum stability remains open; this will not also change considerably if the central value of M_t shifts by some 1 GeV (with implied 1.5 GeV future uncertainty of m_t)

Two-loop electroweak corrections to the top quark mass

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introduction

- One of the puzzles in the SM is the large ratios of fermion masses (e.g. $m_t/m_e \sim 10^5$).
- Many attempts to explain this are based of some underlying structures at higher energy scales (GUT's theories etc.).
- A successful example could be m_b/m_τ, predicted in SUSY GUT's, assuming Yukawa unification at $M_{\rm GUT} \sim 10^{16}$GeV
- The dominant radiative corresctions to such relations can be included via RG evolution. Big progress here, e.g. recently 3-loop RG functions in the SM became available.
- Other significant effects arise at the thresholds, when we try to relate the running parameters to physical observables.
- In contrast to RG calculations (being in fact massless) the matching calculations should take into account mass effects.

introduction

- We need some parameter(s) to describe mass effects.
- Since quarks do not appear as free particles, the concept of a quark mass is not unambiguous. The most often used are "pole" (or "on-shell") mass M and $\overline{\mathrm{MS}}$ running mass $m(\mu)$ defined in $\overline{\mathrm{MS}}$ scheme.
- It is of great importance to establish relaible relations between different mass definitions.
  - scheme dependece of electroweak observables:
    $\longrightarrow$ theoretical uncertainty due to scheme depndece in an observable that depends on top quark mass is proportianal to $m_t(M_t) - M_t$
  - extraction of the quark masses from experimental data:
    $\longrightarrow$ the value extracted by reconstracting the invariant mass of the top quark decay products is expected to be close to M, while the analysis of the $\sigma(\bar{t}t + X)$ yields a clean determination of $\overline{\mathrm{MS}}$ mass

stability of electroweak vacuum

Higg potential in the Standard Model: $V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$

- spontanous breaking of symmetry
  $\longrightarrow$ nonzero value of $v = \langle\phi\rangle \sim 246\text{GeV}$
- quantum corrections modify potential wich could lead to instability
  $\longrightarrow$ bound of the Higgs boson mass from below $M_H^{\text{stab.}}$

Can the SM be extrapolated up to ultimate scales while still having an absolutely stable electroweak vacuum? Cabibbo, Maiani, Parisi, Petronzio '79

Theoretical analysis requires two parts:

- evolution of the running parameters with the renormalization group
- determination of the running parameters from the physical observables

For $\text{N}^{\text{p}}\text{LO}$ evolution we need $\text{N}^{\text{p}-1}\text{LO}$ matching

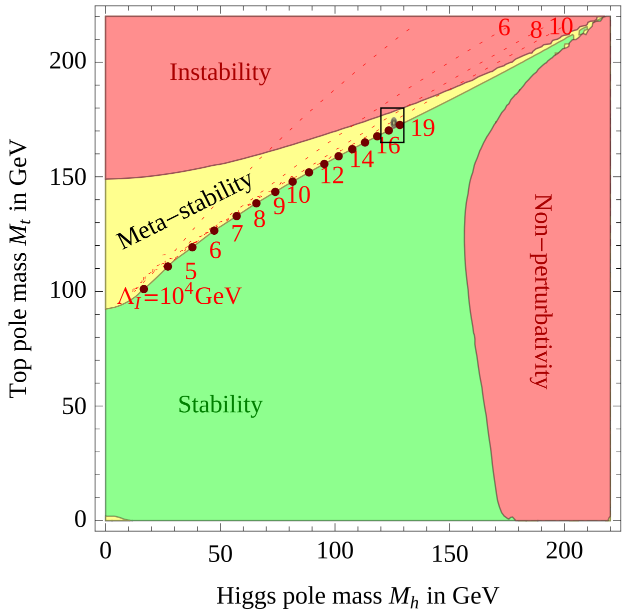
recent theoretical predictions for $M_H^{\text{stab.}}$ (in GeV)

$$126.3 + 4.1\frac{M_t - 171.2}{2.1} - \frac{\alpha_s(M_Z^2) - 0.1176}{2 \cdot 10^{-3}}$$

Bezrukov, Shaposhnikov, Kalmykov, Kniehl '12

$$129.2 + 1.8\frac{M_t - 173.2}{0.9} - 0.5\frac{\alpha_s(M_Z^2) - 0.1184}{7 \cdot 10^{-4}} \pm 1$$

Degrassi, Di Vita, Elias–Miro, Espinosa, Giudice, Isidori, Strumia '13



♠ change in the input value M_t by $\Delta M_t \sim \pm 1$GeV

leads to change of the Higgs bound by $\Delta M_H \sim \pm 2$GeV

the pole mass M_t

Theoretically: M is related to the pole of the fermion propagator $\not{S}(p)$.

Experimentally: Kinematical reconstruction of the invariant mass of the top quark decay products. At Tevatron by CDF and D0 collaborations

$$M_t^{\exp} = 173.2 \pm 0.9\text{GeV}$$

Is it necessarily the pole mass entering, *i.a.*, the stability bound?

Problems:

- intrinsic uncertainty due to non-perturbative mechanizm of hadronization
- intrinsic limitation of the "on-shellness" because of QCD confinetment
- infrared renormalons lead to ambiguity of the order $O(\Lambda_{\text{QCD}})$
- ...

$\longrightarrow$ the uncertainty is hard to quantify

the running mass $m_t(\mu)$

Theoretically: Intrinsically only defined within the perturbation theory; realizes the concept of a running parameter, which depends on the hard scale μ in complete analogy to $\alpha_s(\mu)$.

Experimentally: Determination of $m_t(\mu)$ is possible from the mass dependence of any observable, which is:

- measured precise enough
- theretically predicted beyond the leading order

e.g., total cross-section $\sigma(\bar{t}t + X)$ measured both on Tevatron and LHC (NNLO QCD analysis)

$$m_t^{\rm exp}(m_t) = 163.3 \pm 2.7 \mathrm{GeV}$$

Alekhin, Djouadi, Moch '13

the top mass

$\Longrightarrow$ aiming at ± 1.5GeV accuracy with LHC data.

$$\sigma(\bar{t}t + X) \longrightarrow m_t(\mu) \longrightarrow M_t$$

- NNLO QCD (done)
- least NLO EW
- more LHC data
- NNLO EW shift $m_t(M_t) - M_t$ (this work)

pole of the fermion propagator

◇ fermion propagator:

$$S(\not{p}) = \frac{1}{\not{p} - m_0 - \not{\Sigma}(p)}$$

$$\not{\Sigma}(p) = \not{p}\omega_L A_L(p^2) + \not{p}\omega_R A_R(p^2) + m_0 B(p^2)$$

with $\omega_L = (1-\gamma_5)/2$, $\omega_R = (1+\gamma_5)/2$

◇ no pseudoscalar contribution $\longrightarrow$ CP violation

$$\not{S}(p) = \not{S}_L(p) + \not{S}_R(p)$$

◇ poles of matrices S_L and S_R *coincide*

$$p^2[1 - A_L(p^2)][1 - A_R(p^2)] - m_0[1 + B(p^2)]^2 = 0 \qquad (*)$$

◇ solve Eq. $(*)$ perturbatively

$$p = m_0(1 + X_1 + X_2 + \ldots)$$

pole of the fermion propagator (2)

solution:

$$\begin{aligned} X_1 &= B_1 + \frac{1}{2}A_{L,1} + \frac{1}{2}A_{R,1} \\ X_2 &= B_2 + \frac{1}{2}A_{L,2} + \frac{1}{2}A_{R,2} + X_1(A_{L,1} + A_{R,1} + A'_{L,1} + A'_{R,1} + 2B'_1) \\ &\quad -\frac{1}{2}X_1^2 - \frac{1}{2}A_{L,1}A_{R,1} + \frac{1}{2}B_1^2, \qquad \text{etc} \end{aligned}$$

with $B = B(m_0^2)$, $B' = m_0^2 dB(m_0^2)/dm_0^2$, etc.

- $X_1, X_2, \ldots$ are infrared safe and gauge invariant
- taking into account final width $\longrightarrow$ complex pole

$$p^2 = M^2 - iM\Gamma$$

- renomaliztion in $\overline{\text{MS}}$ scheme

  $\longrightarrow$ relation between M and $m(\mu)$

3-loop QCD analysis

$$
\begin{aligned}
\frac{\overline{m}(M)}{M} = {} & 1 - \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right) + \left(\frac{\alpha_s}{\pi}\right)^2 \left(N_L\left(\frac{71}{144} + \frac{\pi^2}{18}\right) - \frac{3019}{288} + \frac{1}{6}\zeta_3 - \frac{\pi^2}{9}\log 2 - \frac{\pi^2}{3}\right) \\
& + \left(\frac{\alpha_s}{\pi}\right)^3 \left(N_L^2\left(-\frac{2353}{23328} - \frac{7}{54}\zeta_3 - \frac{13}{324}\pi^2\right) + N_L\left(\frac{246643}{23328} + \frac{241}{72}\zeta_3 + \frac{11}{81}\pi^2\log 2 - \frac{2}{81}\pi^2\log^2 2\right.\right. \\
& \left. + \frac{967}{648}\pi^2 - \frac{61}{1944}\pi^4 - \frac{1}{81}\log^4 2 - \frac{8}{27}\mathrm{Li}_2(1/2)\right) - \frac{9478333}{93312} + \frac{1439}{432}\zeta_3\pi^2 - \frac{61}{27}\zeta_3 - \frac{1975}{216}\zeta_5 \\
& \left. + \frac{587}{162}\pi^2\log 2 + \frac{22}{81}\pi^2\log^2 2 - \frac{644201}{38880}\pi^2 + \frac{695}{7776}\pi^4 + \frac{55}{162}\log^4 2 + \frac{220}{27}\mathrm{Li}_2(1/2)\right)
\end{aligned}
$$

1980–2000

Numerically ($a = \alpha_s(M)/\pi$)

$$\frac{m(M)}{M} = 1 - 1.3a + (-14. + 1.n_l)a^2 + (-202. + 27.n_l - 0.7n_l^2)a^3 + \ldots$$

(the $O(\alpha_s^4)$ result is on the way)

For the top quark: $m_t(M_t) - M_t = -10.4\text{GeV}$

What new comes with electroweak corrections?

running masses in the SM

SM parameters:

- symmetric phase parameters: $g, g', y_f, m_\phi, \lambda$
- broken phase parameters: e, m_W, m_Z, m_H, m_f and $G_F = 1/\sqrt{2}v^2$

expectations: spontaneous symmetry breaking does not affect the UV structure of the SM

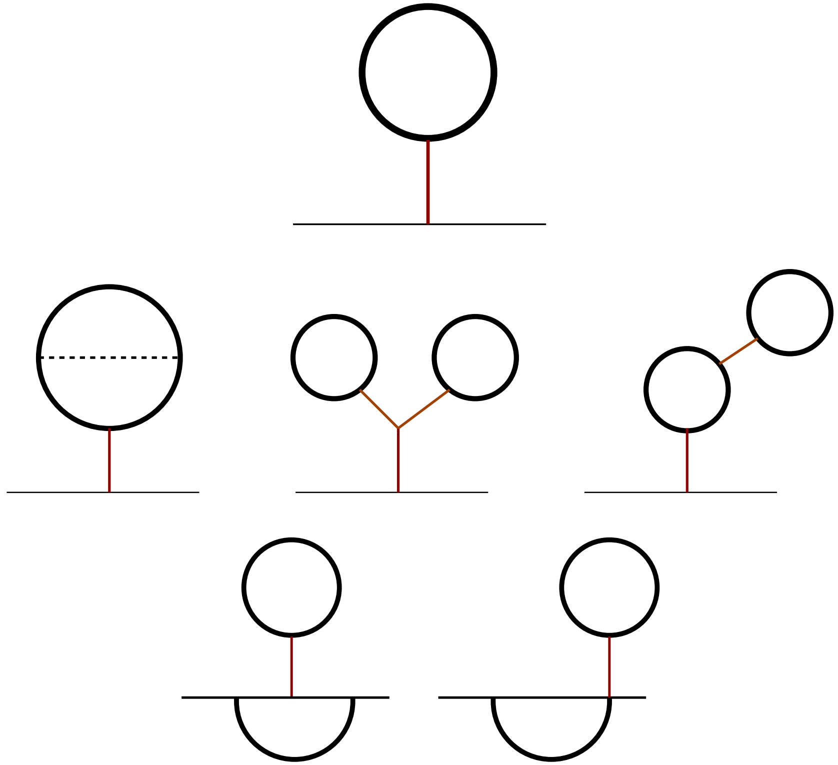
facts (theorem):

$\diamondsuit$ there are interrelations between RG functions of massive parameters in broken phase and the RG functions of the parameters in symmetric phase

requirements:

- inclusion of all diagrams with tadpoles
  $\longrightarrow$ (gauge invariant parametrization of the theory)
- it is not sufficient to select gauge invariant subset
  $\longrightarrow$ (RG equations are different, UV structure is not preserved)

tadpoles



1-loop: $T = \frac{1}{\varepsilon}\frac{g^2}{16\pi^2}\left(2N_c\frac{m_t^4}{m_W^2 m_H^2} - \frac{1}{2} - \frac{3m_W^2}{2m_H^2 c_W^4} - \frac{1}{4c_W^2} + \frac{1}{4c_W^2}(1-\xi_Z) - \frac{3m_H^2}{4m_W^2} + \frac{1}{2}(1-\xi_W) - 3\frac{m_W^2}{m_H^2}\right) + \dots$

- gauge dependent
- large contributions, e.g. $\frac{g^2}{16\pi^2}2N_c\frac{m_t^4}{m_W^2 m_H^2} \sim 0.13$

tadpoles (2)

It is possible to take into account tadpoles by renormalizing v.
Higgs lagrangian

$$L_H = \frac{1}{2} m_0^2 \phi_0^2 - \frac{\lambda}{4!} \phi_0^4$$

- supply a condition

$$v_0(3v_0^2\lambda_0 - m_0^2) = 0$$

consistent with gauge invariance (all bare parameters are g.i.)

- introduce additional renormalization constant

$$v_0 \longrightarrow v_0 Z_v^{1/2}$$

Then $\delta Z_v^{(1)} = \frac{g}{M_W M_H} \Pi_H^{(1)}$,

$$\delta Z_v^{(2)} = \frac{g\,\Pi_H^{(2)}}{M_W M_H} - \frac{\delta Z_v^{(1)}}{2} \left(\delta Z_v^{(1)} + \delta Z_H^{(1)} + \frac{2\delta M_H^{2(1)}}{M_H} + \frac{\delta M_W^{2(1)}}{M_W} + \frac{2\delta s_W^{(1)}}{s_W} - \delta Z_e^{(1)} \right)$$

RG functions functions in the SM

definitions: $\beta_X = \mu^2 \frac{d}{d\mu^2} X, \qquad \gamma_X = \mu^2 \frac{d}{d\mu^2} \ln X$

The evolution of quantity $X(\mu)$ is governed by the renormalization group equation

$$\left(\gamma_X + \sum_j \beta_j \frac{\partial}{\partial g_j} + \sum_i \gamma_j \frac{\partial}{\partial \ln m_i^2} \right) X = 0$$

which should be supplied with corresponding matching conditions.

RG functions functions in the SM

Relations between RG functions of masses and couplings in different phases

$$\gamma_{m_f} = \underbrace{\gamma_{y_f} + \frac{1}{2}\left(\gamma_{m_\phi^2} - \frac{\beta_\lambda}{\lambda}\right)}_{\text{symmetric phase}} \qquad \gamma_{m_W^2} = \underbrace{2\frac{\beta_g}{g} + \left(\gamma_{m_\phi^2} - \frac{\beta_\lambda}{\lambda}\right)}_{\text{symmetric phase}}$$

$$\frac{\gamma_{v^2}}{v^2} = \left(\gamma_{m_\phi^2} - \frac{\beta_\lambda}{\lambda}\right) \qquad \gamma_{G_F} = -\left(\gamma_{m_\phi^2} - \frac{\beta_\lambda}{\lambda}\right)$$

verified also by explicit 2-loop analysis

---

correspondence

symmetric phase	broken phase
quadratic divergences	tadpole diagrams
in the renormalization of m^2	in the renormalization of m_X^2, G_F

RG functions functions in the SM

how it looks

$$
\begin{aligned}
\gamma_{m_t^2} \;=\; & \left(\frac{g^2}{16\pi^2}\right)\Big\{\frac{2}{3} - \frac{3}{4}\frac{m_H^2}{m_W^2} + \frac{3}{4}\frac{m_t^2}{m_W^2} - 3\frac{m_W^2}{m_H^2} - \frac{2}{3}\frac{m_Z^2}{m_W^2} - \frac{3}{2}\frac{m_Z^4}{m_H^2 m_W^2} + 6\frac{m_t^4}{m_H^2 m_W^2}\Big\} \\
+ & \left(\frac{g^2}{16\pi^2}\right)^2\Big\{n_G\Big(-\frac{89}{162} + \frac{16}{3}\frac{m_W^2}{m_H^2} + \frac{64}{9}\frac{m_Z^2}{m_H^2} - \frac{65}{162}\frac{m_Z^2}{m_W^2} + \frac{65}{324}\frac{m_Z^4}{m_W^4} - \frac{80}{9}\frac{m_Z^4}{m_H^2 m_W^2} + \frac{40}{9}\frac{m_Z^6}{m_H^2 m_W^4}\Big) + \frac{11}{48} + \frac{35}{2}\frac{m_t^2}{m_H^2} \\
+ & \frac{33}{32}\frac{m_H^4}{m_W^4} + \frac{3}{4}\frac{m_H^2 m_t^2}{m_W^4} - \frac{69}{32}\frac{m_t^4}{m_W^4} - \frac{15}{2}\frac{m_t^6}{m_H^2 m_W^4} - \frac{3}{4}\frac{m_H^2}{m_W^2} + \frac{91}{96}\frac{m_t^2}{m_W^2} - \frac{4}{3}\frac{m_t^4}{m_H^2 m_W^2} - \frac{176}{3}\frac{m_W^2}{m_H^2} - \frac{17}{2}\frac{m_Z^2}{m_H^2} - \frac{3}{8}\frac{m_H^2 m_Z^2}{m_W^4} \\
+ & \frac{223}{192}\frac{m_t^2 m_Z^2}{m_W^4} + \frac{4}{3}\frac{m_t^4 m_Z^2}{m_H^2 m_W^4} + \frac{2}{3}\frac{m_Z^2}{m_W^2} - 20\frac{m_t^2 m_Z^2}{m_H^2 m_W^2} - \frac{289}{192}\frac{m_Z^4}{m_W^4} + \frac{19}{4}\frac{m_t^2 m_Z^4}{m_H^2 m_W^4} + \frac{31}{12}\frac{m_Z^4}{m_H^2 m_W^2} + \frac{59}{24}\frac{m_Z^6}{m_H^2 m_W^4}\Big\}
\end{aligned}
$$

the running fermion mass

the running fermion mass

At the tree level:

$$m_W = \frac{gv}{2} \qquad y = \frac{gm}{\sqrt{2}m_W} \qquad G_F = \frac{1}{\sqrt{2}v^2}$$

therefore $\quad m = 2^{-3/4}G_F^{1/2}y.$

Possible definitions beyond the LO

$$m(\mu) = 2^{-3/4}G_F(\mu)^{1/2}y(\mu)$$

- the "true" $\overline{\text{MS}}$ mass as it comes from diagram calculus in accordance with $\overline{\text{MS}}$ prescription
- among others includes the tadpole contributions

$$m_Y(\mu) = 2^{-3/4}G_F^{1/2}y(\mu)$$

- at the energy scales below $\sim 2m_W$ effectively the same as $m(\mu)$ due to properties of G_F (see below)
- free of tadpoles as the Yukawa coupling is not affected by Higss mechanism

$$\mu^2\frac{dG_F(\mu)}{d\mu^2} = \frac{G_F(\mu)}{8\pi^2\sqrt{2}\,m_W^2}\left\{\sum_{f\neq t}\left(m_f^2 - 4\frac{m_f^4}{m_H^2}\right) + 2\frac{6m_W^4 + 3m_Z^4 + m_H^4 - 4m_t^4}{m_H^2} - 6m_W^2 - 3m_Z^2 + m_H^2 + 2m_t^2\right\}$$

the fermion mass (2)

It is convinient to reexpress G_F in $\overline{\text{MS}}$ scheme

$$2^{5/2}G_F = \frac{e^2(\mu)}{m_W^2(\mu)\sqrt{1 - m_W^2(\mu)/m_Z^2(\mu)}}(1 + \overline{\Delta r}(\mu)),$$

here G_F id fixed! (e.g. as G_μ)
Then

$$m_Y(\mu) = \frac{m(\mu)}{\sqrt{1 + \overline{\Delta r}(\mu)}}$$

- running as the Yukawa coupling in the symmetric phase
- matching as $\overline{\text{MS}}$ mass at low energies
- free of tadpoles (quadratic divergences) and still being gauge invariant
- at the tree level $M = m(\mu) = m_Y(\mu)$
- in pure QCD $m(\mu) = m_Y(\mu)$

running masses at 2-loops, bottom quark

$$
\begin{aligned}
\frac{m_b(M_b)}{M_b} &= 1+\delta_{\rm QCD}(M_b)+\frac{\alpha}{4\pi}\Big\{N_c\frac{M_t^4}{M_W^2M_H^2S_W^2}(1-L_t)+\frac{M_t^2}{M_W^2S_W^2}(-\frac{5}{16}+\frac{3}{8}L_t)+\dots\Big\}\\
&+C_F\frac{\alpha_s(M_b)}{4\pi}\frac{\alpha}{4\pi}\Big\{N_c\frac{M_t^4}{M_W^2M_H^2S_W^2}(-2+16L_t+3L_b-3L_bL_t-6L_t^2)+\frac{M_t^2}{M_W^2S_W^2}(-\frac{13}{4}-\frac{3}{2}L_t-\frac{15}{16}L_b+\dots)+\dots\Big\}\\
&+\Big(\frac{\alpha}{4\pi}\Big)^2\Big\{\Big(N_c\frac{M_t^4}{M_W^2M_H^2S_W^2}\Big)^2\frac{(1-L_t)(3+L_t)}{2}-\frac{N_cM_t^6}{16M_W^4M_H^2S_W^4}(77-75L_t+18L_t^2+8\zeta_2)\\
&+\frac{N_cM_t^4}{M_H^4}\Big(\frac{-7+5L_t+3L_z+3L_tL_z}{4C_W^4}+\frac{-7+5L_t+3L_z+3L_tL_z}{2C_W^2}+\dots\Big)+\dots\Big\}\\
&= 1+\delta_{\rm QCD}(M_b)-0.42-0.20+\dots
\end{aligned}
$$

$$
\begin{aligned}
\frac{m_{Y,b}(M_b)}{M_b} &= 1+\delta_{\rm QCD}(M_b)+\frac{\alpha}{4\pi}\Big\{N_c\frac{M_t^4}{M_W^2M_H^2S_W^2}(1-L_t)+\frac{M_t^2}{M_W^2S_W^2}(-\frac{5}{16}+\frac{3}{8}L_t)+\frac{M_t^2M_W^2}{(M_t^2-M_W^2)^2S_W^2}(-\frac{3}{8}L_{tw})+\dots\Big\}\\
&+C_F\frac{\alpha_s(M_b)}{4\pi}\frac{\alpha}{4\pi}\Big\{N_c\frac{M_t^4}{M_W^2M_H^2S_W^2}(-2+16L_t+3L_b-3L_bL_t-6L_t^2)+\frac{M_t^2}{M_W^2S_W^2}(-\frac{13}{4}-\frac{3}{2}L_t-\frac{15}{16}L_b+\dots)+\dots\Big\}\\
&+\Big(\frac{\alpha M_t^2}{4\pi M_W^2S_W^2}\Big)^2\Big\{A_{2,2}\ln^2\frac{M_t^2}{M_b^2}+A_{2,1}\ln\frac{M_t^2}{M_b^2}-\frac{45}{128}\frac{M_t^4}{M_H^4}\ln^2\frac{M_t^2}{M_H^2}+\Big(-\frac{5}{16}+\frac{241}{128}\frac{M_t^2}{M_H^2}+\frac{45}{32}\frac{M_t^4}{M_H^4}\Big)\ln\frac{M_t^2}{M_H^2}+\dots\Big\}.\\
&= 1+\delta_{\rm QCD}(M_b)-0.0197-0.0068+\dots
\end{aligned}
$$

running masses at 2-loops, bottom quark

$$
\begin{aligned}
\frac{m_b(M_b)}{M_b} &= 1+\delta_{\rm QCD}(M_b)+\frac{\alpha}{4\pi}\Big\{N_c\frac{M_t^4}{M_W^2 M_H^2 S_W^2}(1-L_t)+\frac{M_t^2}{M_W^2 S_W^2}(-\frac{5}{16}+\frac{3}{8}L_t)+\dots\Big\}\\
&+C_F\frac{\alpha_s(M_b)}{4\pi}\frac{\alpha}{4\pi}\Big\{N_c\frac{M_t^4}{M_W^2 M_H^2 S_W^2}(-2+16L_t+3L_b-3L_bL_t-6L_t^2)+\frac{M_t^2}{M_W^2 S_W^2}(-\frac{13}{4}-\frac{3}{2}L_t-\frac{15}{16}L_b+\dots)+\dots\Big\}\\
&+\left(\frac{\alpha}{4\pi}\right)^2\Big\{\left(N_c\frac{M_t^4}{M_W^2 M_H^2 S_W^2}\right)^2\frac{(1-L_t)(3+L_t)}{2}-\frac{N_cM_t^6}{16M_W^4 M_H^2 S_W^4}(77-75L_t+18L_t^2+8\zeta_2)\\
&+\frac{N_cM_t^4}{M_H^4}\Big(\frac{-7+5L_t+3L_z+3L_tL_z}{4C_W^4}+\frac{-7+5L_t+3L_z+3L_tL_z}{2C_W^2}+\dots\Big)+\dots\Big\}\\
&= 1+\delta_{\rm QCD}(M_b)-0.42-0.20+0.37
\end{aligned}
$$

$$
\begin{aligned}
\frac{m_{Y,b}(M_b)}{M_b} &= 1+\delta_{\rm QCD}(M_b)+\frac{\alpha}{4\pi}\Big\{N_c\frac{M_t^4}{M_W^2 M_H^2 S_W^2}(1-L_t)+\frac{M_t^2}{M_W^2 S_W^2}(-\frac{5}{16}+\frac{3}{8}L_t)+\frac{M_t^2M_W^2}{(M_t^2-M_W^2)^2S_W^2}(-\frac{3}{8}L_{tw})+\dots\Big\}\\
&+C_F\frac{\alpha_s(M_b)}{4\pi}\frac{\alpha}{4\pi}\Big\{N_c\frac{M_t^4}{M_W^2 M_H^2 S_W^2}(-2+16L_t+3L_b-3L_bL_t-6L_t^2)+\frac{M_t^2}{M_W^2 S_W^2}(-\frac{13}{4}-\frac{3}{2}L_t-\frac{15}{16}L_b+\dots)+\dots\Big\}\\
&+\left(\frac{\alpha M_t^2}{4\pi M_W^2 S_W^2}\right)^2\Big\{A_{2,2}\ln^2\frac{M_t^2}{M_b^2}+A_{2,1}\ln\frac{M_t^2}{M_b^2}-\frac{45}{128}\frac{M_t^4}{M_H^4}\ln^2\frac{M_t^2}{M_H^2}+\left(-\frac{5}{16}+\frac{241}{128}\frac{M_t^2}{M_H^2}+\frac{45}{32}\frac{M_t^4}{M_H^4}\right)\ln\frac{M_t^2}{M_H^2}+\dots\Big\}.\\
&= 1+\delta_{\rm QCD}(M_b)-0.0197-0.0068+0.0057
\end{aligned}
$$

running masses at 2-loops, top quark

The various contributions to $m_t(M_t) - M_t$ (in GeV).

Kalmykov, Jegerlehner, Kniehl

Kniehl, OV

M_H [GeV]	QCD	$O(\alpha)$	$O(\alpha\alpha_s)$	$O(\alpha^2)$	total
124	−10.38	12.11	−0.39	−0.51	0.83
125	−10.38	11.91	−0.39	−0.49	0.65
126	−10.38	11.71	−0.38	−0.48	0.46

Large cancellations between QCD and EW corrections!